studied by varying the concentration N. Such studies have been carried out by Smith, *et al.*[14]

Conclusion

A variety of phenomena involving the behavior of a plasma in a solid in the presence of a magnetic field has been described. With almost no exception, each of these in one form or another reflects the band properties of the semiconductor or metal which is being investigated. Although the phenomena are complex and of primary interest to physicists who wish to measure the fundamental parameters associated with holes and electrons in these materials, the results are, nevertheless, already of some practical significance to engineers who wish to utilize these effects for developing new kinds of devices. Obviously, these magnetoplasma effects can be utilized as polarizers and for nonreciprocal components in the regions of the far infrared spectrum where such devices do not exist. However, this type of investigation is attractive because it may be instrumental in the development of an infrared cyclotron resonance maser. The magnetoplasma effects permit not only the investigation of the anisotropy of the effective masses, but also their variations with energy, an important requirement for the development of such a cyclotron resonance maser.[15]

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Coherent Excitation of Plasma Oscillations in Solids*

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Summary—**Considerations are put forth concerning the feasibility of observing the coherent excitation of plasma oscillations in a two-component plasma of electrons and holes in semiconductors or semimetals. By coherent excitation is meant the onset of a high-frequency ("two-stream") instability arising from an appreciable drift of electrons vs holes under the action of an applied electric field. Conditions favorable to coherent excitation include a sizeable difference in electron and hole masses, and long relaxation times for both kinds of particles. The extent to which such conditions are present in InSb is discussed.**

THE PLASMA formed by the electrons and holes in a semiconductor or semimetal offers, in many respects, a promising "laboratory" for carrying out experiments of interest on collective properties of plasmas. By gaseous standards, the plasma is well behaved. One can measure and vary in simple fashion the concentrations, mass ratios, and temperature ratios of the two plasmas. The principal drawback to carrying out plasma experiments is that the electrons and holes in this fully ionized plasma are not completely free; they are scattered by phonons, impurity atoms, and, in some cases, one another, to an extent which may be important for the study of collective phenomena. Indeed, if ω is the frequency of interest for the phenomenon under investigation, and $\tau_{\pm}$ represents the electron (or hole) relaxation time associated with the scattering mechanisms, then it is necessary that

$$\omega \tau_{\pm} \gtrsim 1,$$

in order that the collective behavior be observable.

In the present paper some theoretical investigations of collective behavior in solid-state plasmas, which have been carried out in collaboration with J. R. Schrieffer,[1] will be summarized. The problem of particular concern was the feasibility of observing in such plasmas a high frequency instability associated with the coherent excitation of plasma oscillations. The instability, which resembles the "two-stream" instability encountered in electron beam studies, arises if a sufficiently large drift of electrons vs holes is produced under the action of an applied electric field.

Most previous studies[2] of instabilities in the two component plasmas were carried out under the assumption that the electron and ion (or hole) temperatures were equal. In these circumstances the required drift velocity is of the order of $1.3v_-$, where v_- is the electron thermal

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velocity, defined by $mv_-^2 = \kappa T_-$. However, when the electron-ion temperature ratio, T_-/T_+, is sufficiently large, the critical drift velocity may be reduced below v_-, to a value of the order of $(m_-/m_+)^{1/2}\, v_-$,[3] where $m_\pm$ are the electron and hole effective masses. It does not appear out of the question to produce drift velocities of this latter order of magnitude in a semiconductor, so that one is led to investigate closely the question of instabilities for a plasma in which $T_- \gg T_+$.

There are two aspects to the problem. First, it is desirable to calculate the conditions for the existence of instabilities, and their associated growth rates, for a considerable range of electron and hole effective mass and temperature ratios, and their net drift velocity v_d. Such a calculation is carried out by solving the dispersion relation for the plasma oscillations in the solid. The dispersion relation tells us the allowed frequencies ω, for an oscillating longitudinal mode of wave-vector k; that is, the conditions for a potential wave, of the form

$$\phi_k \exp i(\boldsymbol{k}\cdot \mathbf{x} - \omega t)$$

to exist in the solid. The frequency ω is in general complex,

$$\omega = \omega_1 + i\omega_2.$$

Where $\omega_2 < 0$, the oscillation is damped, and where $\omega_2 > 0$, the oscillation will grow, and an instability exists; $\omega_2 = 0$ defines the boundary between growing waves and damped waves.

The second aspect concerns the possibility of achieving, in practice, the desired range of effective masses, concentrations, $n_\pm$, temperature ratios, and drift velocities, such that a certain growth rate ω_g for the instability might be expected. Here, one attempts to solve the "hot electron" problem to determine the extent to which a given electric field will act to produce heating of the electrons and holes, plus a net drift velocity of electrons vs holes, in the presence of a variety of external scattering mechanisms.

The principal assumption which is made in obtaining the dispersion relation for the plasma oscillations is that there exist two reasonably distinct time scales which characterize the change in the velocity distribution function $f_\pm(\boldsymbol{r}, \boldsymbol{v}, t)$, for the electrons and holes. The first time scale is associated with the macroscopic drift and heating of the particles induced by the external electric field. The second time scale, which is assumed short compared to the first, is that which characterizes the coherent behavior of the system, as manifested by the collective oscillations and their growth rate (where it exists). In these circumstances, the plasma dispersion relation may be obtained from the collisionless Boltzmann equation in which particle interaction is taken into account by means of a self-consistent field,[4] a procedure which is equivalent to the random phase approximation.[5] The dispersion relation may be written as

$$1 = \frac{4\pi e^2}{\epsilon_0 m_+ k^2}\int d\boldsymbol{v}\, \frac{\boldsymbol{k}\cdot\nabla_v f_+}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + i\delta} + \frac{4\pi e^2}{\epsilon_0 m_- k^2}\int d\boldsymbol{v}\, \frac{\boldsymbol{k}\cdot\nabla_v f_-}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + i\delta} \tag{1}$$

where, for simplicity, we assume that the $f_\pm$ take the form of displaced Maxwellian velocity distributions

$$f_\pm(\boldsymbol{E}_0) = a_\pm \exp - \frac{m_\pm(v - v_{d\pm})^2}{2kT_\pm},$$

which will be the case if the collisions between the particles are more effective in relaxing their momentum and energy distributions than their interactions with phonons, impurities, etc. Here $v_{d\pm}$ and $T_\pm$ are the drift velocities and temperatures brought about by the external field E_0; it is these quantities which are assumed to change adiabatically with regard to the times characteristic of plasma effects. ϵ_0 is the static dielectric constant of the semiconductor, and the small imaginary part $i\delta$ is introduced to furnish the appropriate prescription for integrating around the poles at $\boldsymbol{k}\cdot\boldsymbol{v} = \omega$.

The results of the calculations for the boundary between growing waves and damped waves, and the growth rates associated with the low-frequency plasma wave instability are presented in Figs. 1 and 2. The calculations have been carried out for $m_+ = 14m_-$, a ratio which is appropriate for InSb, one of the most promising semiconductors in which to study such instabilities. The dispersion relation (1) has been solved for a coordinate system in which the holes have zero drift velocity, and the electron drift velocity is $v_d = v_{d_-} - v_{d_+}$. Furthermore both τ_+ and τ_- have been taken as infinite. The low-frequency plasma modes under consideration are the so-called acoustic modes, in which the electrons and holes oscillate in phase. In Fig. 1 is plotted the boundary between growing waves and damped waves for several values of the ratio T_-/T_+. Below (or to the right of, in the case $T_+ = 0$), a particular curve, one has growth, while above (or to the left), one has damping. In Fig. 2 there appear the growth rate curves for three cases of interest. The growth rate ω_2 is measured in units of the hole-plasma frequency, $\omega_+ = (4\pi n_+ e^2/m_+\epsilon_0)^{1/2}$, while the wave-vector k is measured in units of the electron Debye wave-

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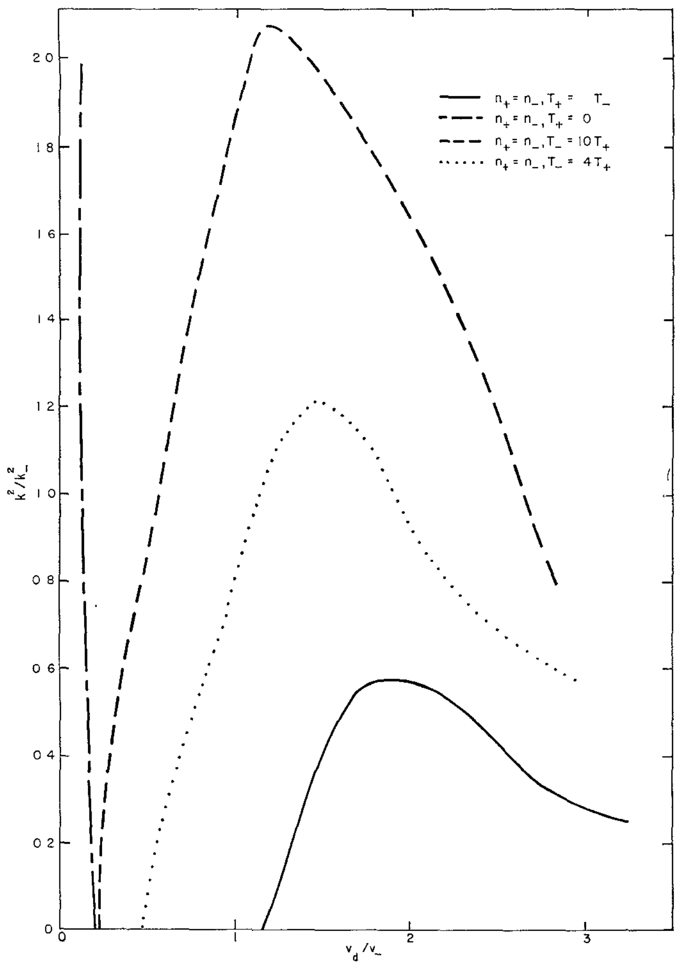

Fig. 1—The boundary between growing waves and damped waves for an electron-hole plasma with $n_+ = n_-$, $m_+ = 14\ m_-$, and varying values of T_-/T_+.

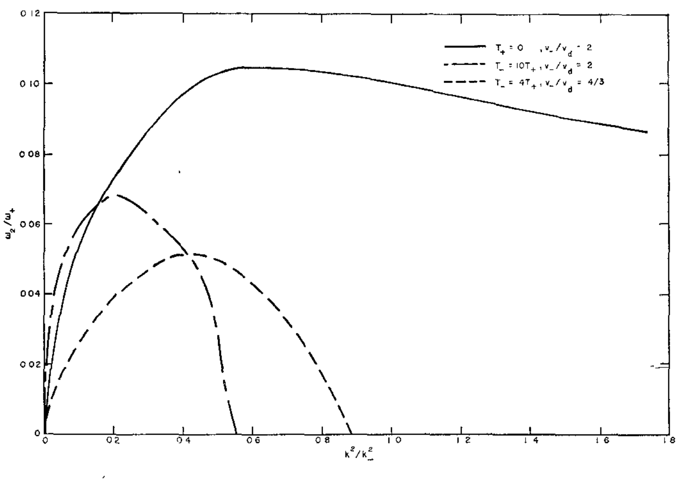

Fig. 2—Growth rate curves for several cases of interest.

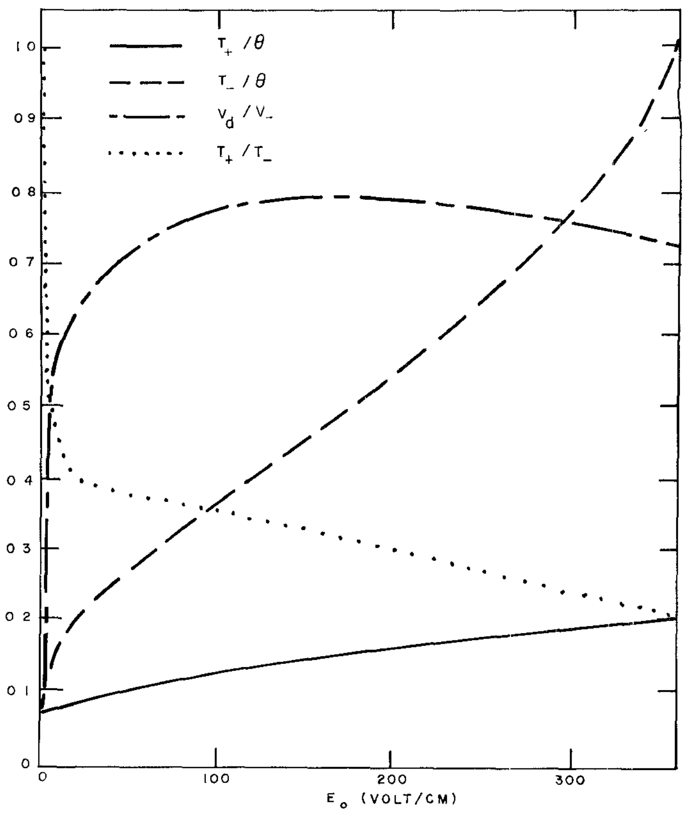

Fig. 3—Hole and electron temperatures and electron drift velocity as a function of field strength in InSb, assuming the following initial conditions: $T_{\text{lattice}} = 20°$K; $m_- = 0.03m$; $m_+ = 0.18m$; Ni $= 10^{15}$/cc.

vector $k_- = (4\pi n_- e^2/\epsilon_0 T_-)^{1/2}$.

We remark that as the ratio, $T-/T+$, increases, the drift velocity required to produce an instability is reduced toward $\sqrt{m_-/m_+}v_-$. We note further that the growth rates associated with reasonable temperature ratios and values of v_d/v_- tend to be of the order of 5 per cent to 10 per cent of the hole-plasma frequency.

In Fig. 3 are shown the results of some studies of the hole and electron temperatures, and the electron drift velocity, as a function of field strength in InSb.[6] These show that substantial temperature ratios T_-/T_+ and drift velocities v_d/v_- may be achieved with moderate electric fields. By comparing Fig. 3 with Fig. 2 we see that growth rates of the order of $\omega_+/15$ appear achievable, provided the hole and electron scattering by the lattice is negligible. In the present case, the electron scattering turns out to have little effect on the dispersion relation or growth rate. Hole scattering is, however, quite important, and acts to oppose directly the growth of a low-frequency plasma mode. The net growth rate is, thus,

$$\omega_g' = \omega_g - 1/\tau_+$$

where ω_g is the growth rate calculated with neglect of particle-lattice scattering. Hence, a growing wave will be observed only where τ_+ is sufficiently long that

$$\omega_+\tau_+ \geq 15 \qquad (2)$$

for the cases under consideration. A survey of the experimental relaxation times for the holes in InSb shows

[6] For a discussion of the way in which these results are obtained, see Pines and Schrieffer.[1]

that if one can produce $\sim 10^{15}$ electrons and holes per cc, while keeping the ionized impurity concentration below $\sim 10^{15}$/cc and the initial lattice temperature below 20°K, then the condition (2) may be met.

Assuming that the threshold for producing an instability is achieved, what happens next would seem an open question, and a most interesting one. Indeed, our lack of knowledge concerning the subsequent development of an instability is one of the prime reasons for attempting experiments in this area. In this region the behavior of the system is determined by nonlinear effects, such as the coupling between plasma modes of different wavelength. It is possible to give arguments which tend to show that the relative drift velocity will saturate near the threshold for the coherent excitation of the plasma modes, and it is clear that in time the amplitude of these modes will increase substantially over their thermal level of excitation. However, the detailed dynamic behavior of the system is not at all understood and it is, in my opinion, highly desirable that further theoretical and experimental investigations be carried out in this direction.

Pulsed Millimeter-Wave Generation Using Ferrites*

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Summary—**A method is described for generating pulsed RF energy in the millimeter-wave spectrum. Low-loss garnets are used in the uniform precessional mode to store energy at *S* band and radiate at a higher frequency, which is controlled by the total magnetic field. Details are given of a *K*-band generator which operates at frequencies up to 32 kMc.**

Introduction

There has been interest for some time in the possibility of using ferrites in simple solid-state RF generators because ferrites have a high density of electron spins, which at room temperature can store magnetic energy and transform this energy into coherent electromagnetic energy in the microwave and millimeter-wave spectrum. A number of studies[1–5] have been reported of schemes in which the energy would be supplied from a pulsed magnetic field and radiated by the ferrite in the form of short RF pulses. These investigations have disclosed both basic and technological difficulties which are inherent in the problem. These are summarized in an earlier paper on this subject by the present authors.[6]

Some recent experiments with a specific form of pulsed ferrite generator which has proved workable up to 32 kMc are described below. The basic theory and general performance characteristics to be expected from such devices are given in the earlier paper.[6] Briefly, an RF input signal is applied to an yttrium iron garnet (YIG) sphere which is adjusted for gyromagnetic resonance by means of a steady magnetic field, thus establishing a uniform precession. A pulsed magnetic field is applied along the same direction as the above dc field, thereby increasing the resonant frequency of the spins and adding energy to the spin system. During the flat top of this field pulse, the energy stored in the spin system is radiated into a coupled microwave circuit at the new higher frequency. In this type of solid-state generator then, the output frequency is higher than the input frequency; it is not harmonically related to the input frequency, and it may be varied continuously by adjusting the magnitude of the pulsed magnetic field.

The first reports of successful pulsed generation using ferrites are contained in the preceding article[6] and in

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